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*On the Terms upon which the Business of one Insurance Company may be equitably transferred to another. By T. B. SPRAGUE, M.A., F.I.A., Fellow of St. John's College, Cambridge.*

[Read before the Institute of Actuaries, 29th March, 1858, and ordered by the Council to be printed.]

IN treating of the above subject, it cannot be expected that any new principles will be put forward, or that any great amount of novelty will be found in the method of handling the question. All that I have aimed at is, to apply acknowledged principles to the discussion of a question of considerable practical interest at all times, and more peculiarly so at the present time.

It will be admitted, that the basis of the contract for the transfer of the policies of a Company (A) to another (B), must be the liability of (A) under those policies, as shown by the common method of valuation; but when we come to estimate this liability in practice, the question at once arises, whether, in making the valuation, we should value the net premiums upon the various policies, or whether we shall be justified in valuing the gross premiums actually payable. Let the value of the sums assured, calculated by such data as may be agreed upon, be denoted by  $\Sigma$ ; let the value of the net premiums be  $\Pi$ , and that of the actual premiums  $\Pi'$ : then the difference  $\Pi' - \Pi$  is the value of the loading, or of the margin that is added to the net premiums to provide for

expenses and contingencies. If the liability of (A) is taken at  $\Sigma - \Pi$ , the value of this loading would not be anticipated, or the fund for future expenses would be untouched; and on the other hand, if the liability were taken to be only  $\Sigma - \Pi'$ , then the whole of the margin would be anticipated; and if (B) took the business on these terms, it would in effect undertake to manage the business of (A) without charging anything for the necessary expenses of management.

Now a Company, when in negotiation for a transfer of its business, will seldom be in a position to pay the full sum of  $\Sigma - \Pi$ ; and the existing competition between different Insurance Companies will enable it to find some respectable Company to take it upon much easier terms; so that (B) cannot insist on receiving from (A) the full amount  $\Sigma - \Pi$ . On the other hand, it cannot well be considered safe to accept so small an amount as  $\Sigma - \Pi'$ ; nor fair to those already assured, whose profits must be diminished by the expense attending the transaction of the business of (A), when no provision is made in the terms of transfer for that expense. Our conclusion then is, that the liability of (A) should be estimated at some amount intermediate to  $\Sigma - \Pi$  and  $\Sigma - \Pi'$ ; and it will be my endeavour, in the remarks that follow, to point out some of the considerations by which the exact amount intermediate to those two may be fixed, having due regard to what is fair to both parties interested.

It will be at once apparent that the terms granted by (B) to (A) will very much depend upon the relative position of the two Companies. In effect, if (A) is a vigorous Company, with a fair amount of new business, active agents, &c., its connexions will be worth much more to (B) than if it were an old concern which had lost all its vitality, and of which the new business had gradually dwindled down to a mere nominal amount. To fix the ideas, we will suppose that (A) is a Company which finds it impossible to carry on its business with adequate profit, and is anxious to obtain from (B) as favourable terms as possible; and on the other hand, that its connections are so far valuable, that (B) will be willing to undertake the business upon terms which will admit of no direct profit, and at the same time expose (B) to no direct loss. On these suppositions, it is clear that (B) should deduct from the value of the loading,  $\Pi' - \Pi$ , only so much as will be required to provide for expenses actually incurred. One of the most important charges to be provided for is the commission payable—which may be regarded, indeed, simply as a deduction from the premium payable.

For policies on the participating scale, a considerable deduction must be made for the profits to be hereafter declared on the policies; but as this is a distinct consideration, and introduces much more complication, we will suppose for the present that all the policies are on the non-participating scale. The commission paid is usually 5 per cent; of course, if more is paid, it should be taken into account. Probably 5 per cent more will cover the necessary expenses of management; so that, in the ordinary case, a deduction of 10 per cent from the gross premium will be fair to both parties, and the liability will thus be fixed at  $\Sigma - \frac{1}{10} \Pi'$ .

Of course, all compensation to be paid by (B) to the officials of (A), must be added in the account to the liability of (A) which (B) undertakes.

In what precedes, we have supposed the valuation to be made in the ordinary way, which assumes that all the policies will be continued in force; but it is very well known that this is never the case. In the very best Offices, experience shows that some of the policies will be dropped; and in the case of the transfer of the business of a Company, the number of policies dropped will probably always be increased.\* Now when the gross premium is valued, or a deduction of say 10 per cent is made from it, it is well known that for several years the liability comes out negative, or the value of the future premiums is greater than that of the sums assured; and to such an extent is this the case, that it will often happen when an Office has been in existence only a few years, that the total liability under all its policies is negative; and, regarding this circumstance alone, it would have to receive money instead of paying, when it transfers its business. But since it is quite optional for the assured to continue their policies, and as we know that some of them will certainly be discontinued, it seems only right that we should make some allowance on this score. In fact, if we do not, every recent policy dropped after the transfer of (A) to (B), will be a loss to (B); and taking an extreme case, if all the policies in which the liability is negative are dropped, (B) may sustain a very heavy loss by taking the business on such terms. It will be useful to show how to ascertain the amount of the loss which (B) may sustain from this cause. For this purpose, suppose the existing policies divided into classes, containing respectively those in which the liability is positive and those in which it is negative—the gross premiums being valued with the deduction

\* In a mathematical note appended to this paper, I have given a general expression for the value of a policy, when the probability of the policy being dropped is taken into account.

agreed upon, say 10 per cent; then all the policies of the latter class, viz., those in which the liability is negative, appear as an asset in the valuation of the liability of (A). But any one of these policies may be dropped, and in that case the assets of (A) are diminished by the (negative) value of the policy dropped; and supposing all the policies in the latter class to be dropped after the transfer to (B), the assets of (A) transferred to (B) would be less than the liability, by the total amount for which those policies were set down as an asset—or, in other words, (B) would sustain a loss to that extent.

It follows from this, that all policies on which the liability is negative should be taken as *of no value*, in order for (B) to make itself safe; if this is done, and the policies are continued in force, of course there will be a profit. If this extreme course is not taken, it seems only fair that *some* allowance should be made for the risk of loss from this source. For this reason, it cannot be thought too much for (B) to stipulate that all (A)'s policies on the half-credit plan, and those on the ascending scale, in the early stages, should be reckoned as having no value. In fact, there is less probability of such policies being continued in force than those effected on the usual scale; and as they have a larger negative value than the latter, the loss from their being suffered to lapse would be larger. In estimating the amount of the allowance to be made for the reason above stated, a number of considerations are introduced to which it is impossible to attach a precise numerical measure. It will be necessary to consider whether the reputation of the Company (B) is such as to shake the confidence of the policyholders of (A); also, whether there is likely to be any dissatisfaction and opposition from influential parties interested in (A). It is said that an instance has recently occurred, in which some parties, being dissatisfied with what they termed the sale of their Company to another (B), induced a considerable proportion of the assured to transfer their policies to a different one, so that (B) of course was a considerable loser by the transaction.

It is not uncommon, I believe, to provide that the manager of the Company (A) shall exert his influence to obtain the effectual transfer of all the policies to (B); and as in all cases of transfer the manager obtains some compensation, it might be advisable to make the amount of that compensation depend directly upon the effectual transfer of the policies. Thus, for example, he might be allowed a commission on all the policies transferred, subject to capitalization upon equitable terms, at the expiration of such a

time (say, two years) as would show the result of the proposed transfer.

These considerations show that (B), in taking the business of (A) upon practicable terms, by which I mean, such terms as may be agreed upon by both parties, considering the existing competition between different Companies, in fact enters into a kind of speculation. On the one hand, if all goes well, and the majority of the policies are continued, (B) will increase its income and connections, and obtain a staff of agents more or less active and therefore valuable; on the other hand, if from any cause the recent policies of (A) are not continued, (B) is exposed to a direct loss, the amount of which may be serious. In the ordinary case of a transfer being carried out amicably, we may consider the losses from the lapse of recent policies as to some extent compensated by the gain from the new business introduced by the agents of (A); but the same causes which would lead to the discontinuing of the old policies, will operate to prevent the agents of (A) procuring new business for (B). In one instance, I have been told the agents of (A) declined to act for (B), because the commission allowed was not so large as they had been accustomed to. If we wish to take the value of the agents more directly into account, we may reason, that since a Company would consider itself quite justified in spending the first year's income to obtain new business, the agents may roughly be reckoned as worth one year's purchase of the new premiums they introduce.

The division of the policies above indicated into classes in which the liability will be respectively positive and negative, is very easily effected when the premiums charged and the data of valuation are assigned; but it is impossible to say beforehand for how many years after a policy is effected the liability under it will be negative. Suppose that a policy is effected at the age  $m$ , and the premium charged is  $\pi_m$ ; then, deducting 10 per cent from the premium, the value of the policy at the expiration of  $n$  years will be (using the ordinary notation)  $A_{m+n} - \frac{9}{10} \pi_m (1 + a_{m+n})$ , which is equal to  $(p_{m+n} - \frac{9}{10} \pi_m)(1 + a_{m+n})$ , and is therefore positive or negative according as  $p_{m+n}$  (the net premium at age  $m+n$ ) is greater or less than  $\frac{9}{10} \pi_m$ . If we have a table giving the values of  $p_{m+n}$ , the greatest value of  $n$  for which the liability is negative, is seen on inspection.

The rates of premiums charged by different existing Companies vary in a very remarkable manner. In the case of participating rates, a compensation may be effected by the operation of the bonuses allotted, which may be expected to be larger in proportion

as the rates charged are higher. The difference, however, is nearly as great in the non-participating rates. To illustrate this difference, I have selected from a number of prospectuses the highest and lowest non-participating rates charged at certain ages by existing Companies, which are exhibited in the subjoined table.

Age.	Highest rate.	Lowest rate.	Names of the Companies charging these rates respectively.
	£. s. d.	£. s. d.	
15	1 14 9	1 7 8	Rock—International.
20	1 17 4	1 11 0	Mid. Counties—Life Ass <sup>n</sup> of Scotland.
25	2 1 4	1 14 10	Minerva—Unity.
30	2 6 9	1 19 7	Legal and General—Householders'.
35	2 13 7	2 5 6	Ditto ditto.
40	3 1 11	2 13 5	Ditto ditto.
45	3 12 7	3 2 5	Ditto Provident.
50	4 9 1	3 15 3	West of England—British Industry.
55	5 8 0	4 10 4	Alliance—Provident.
60	7 4 0	5 15 4	Provident Clerks'—Life Ass <sup>n</sup> of Scotland.

It is also worthy of remark, that the non-participating rates of some Companies are higher than the participating rates, at the same ages, of other Companies of undoubted stability.

One conclusion to which the previous remarks obviously lead, is, that it is extremely doubtful whether a Company (B) can under any circumstances afford to pay over money to (A) for the transfer of its business. In some recent cases of transfer, I believe one year's income from premiums has been given to the transferring Company. This probably has arisen from a mistaken analogy with fire insurance. But there is the fundamental difference between fire and life insurance, that in the former case the premium charged covers merely the risk for the year, and at the expiration of the year the liability under the policy is at an end, and the contract may be put an end to by either party to it; while in life insurance, the premium charged is much higher than that for the term insurance, and there is a continually growing liability under each policy. In other words, a life policy has a "surrender value."

It will be unnecessary here to do more than point out the very unsatisfactory nature of the above method of adjusting the terms of a transfer.

By valuing the gross premium, in determining the liability of (A), we produce an important effect on the surrender value of its policies. If (B), after the transfer of the business, allows a larger surrender value for any policy than the reserve made in the valuation for it, it incurs a direct loss to the extent of the difference.

While, if the surrender value of the policies is regulated by the reserve made for each, it will follow that the recent policies for several years will have no surrender value at all, and for many years will have a far smaller one than what, by the practice of Offices, is considered a fair value. We shall have also the following rather anomalous conclusion,—that the higher the premiums charged by (A) are, the smaller will be the surrender value allowed on its policies by (B) after the transfer. If any large number of policies were to be surrendered, (B) by giving a fair surrender value might suffer a serious loss, while by giving only such as the terms of transfer warrant, it runs the risk of gaining the character of an illiberal Office. This is a case where opinions will probably be divided as to the best course to pursue. A similar difficulty arises, when a Company which has a very small reserve is asked to purchase its own policies.

It does not seem that any allowance can be well made in the terms of the transfer for this reason; but we see that it is another cause that may diminish (B)'s profit. This bears out the remark made above, that the transaction is a kind of speculation. In order to render itself secure from loss, (B) might stipulate that the liability under the policies of (A) should be taken as equal to the surrender value, in the policies where this value is greater than the reserve made for the policy by the method of valuation agreed upon; and for this purpose, it would probably be sufficient to estimate the surrender value roughly at one third of the premiums paid.

Thus it would appear, that if (B) insists on securing itself from loss, the policies would be divided for valuation into three classes. The first class will contain those policies which have no surrender value, and in which the result of valuing the gross premium is to give a negative value to the policies;—these are to be taken as having no value. The second class contains those which have a surrender value, and in which this value is greater than the reserve made by valuing the gross premium; in these, the surrender value is to be taken as the value of the policy. The third class contains the older policies, in which the surrender value is less than the reserve made by the process of valuation agreed upon; and in these, the latter is to be taken as the value of the policy. The reason why the same process is not applied to the whole of the policies is, in brief, that the option of continuing the policies rests with the assured and not with the Office. It may be doubted whether (B) would, at the present time, be able to insist upon such rigorous terms as have been indicated above.

I come now to the class of participating assurances. As the assured will of course expect to share in the future divisions of profits of (B), it will be necessary to make some provision for the bonuses to be declared, or to make a larger deduction from the gross premiums than is the case in the non-participating assurances. The participating rates are of course higher than the non-participating, but, in general, not sufficiently so to compensate for the amount of the bonuses declared; in fact, as is well known, the profits of an Insurance Company upon the non-participating assurances, supposing the mortality equally favourable, are much larger than upon the participating class. The participating rates seem generally to be, on the average, 10 to 12 per cent higher than the non-participating: in one instance I have noticed, the difference is as much as 20 to 24 per cent at the lower ages; but this, I think, is unusual. However, if the participating policies in (A) are not numerous, it may approximately be sufficient to suppose that the premiums payable on them are at the non-participating rates, and to value them along with the other policies. This will, of course, be most applicable when the difference between the profit and non-profit rates is considerable. If this approximation is not considered sufficient, we may, in some cases, arrive at a fair result by subtracting a percentage of say 25 per cent from the premiums payable, including 5 per cent for commission, and 20 per cent for profits and expenses, which will probably be thought moderate.

But a more correct solution of the question as to the proper deduction to be made from the gross premiums, may be obtained by availing ourselves of the results of Mr. Jellicoe's paper, read before the Institute at its last meeting. It will, I suppose, in all cases be expected that the assured in (A), will after the transfer participate on equal terms with those in (B). In order that they may do so without any loss to the assured of (B), there should be a provision made for the profits on (A)'s policies proportional to that which is made in (B). Thus, if (B) makes a reserve of 30 per cent on the net premiums for the participating policies, the same reserve of 30 per cent should be made on the net premiums of the participating policies in (A), and the corresponding deduction made from the gross premiums. If the sum reserved to provide bonuses for the policies of (A) fall short of what is thus determined, the profits of the assured in (B) will be diminished to that extent, or, more strictly speaking, by such an amount as must be taken from their reserve and added to that of (A), in order to render the two proportional.

One conclusion that may be drawn from what precedes, is, that a Company which makes a small reserve for its future profits will be able, with justice to its own assured, to offer easier terms to a Company desirous of transferring its business than another would which makes a larger reserve.

Nothing has been yet said as to the rate of interest which is to be used in valuing the policies of (A). It is not to be supposed that (B) would use a higher rate in valuing the policies of (A) than it uses for its own valuations; and hence a Company which values at a higher rate of interest would be able to offer easier terms than one which values at a lower rate.

In all that precedes, I have gone upon the supposition that the most liberal terms are offered to (A) by (B), consistent with the condition that (B) is to be exposed to no direct loss. How far a Company, in taking the business of another, may with advantage to itself go beyond this, or in fact *pay* for the increase to its income and connexions, is a point upon which, probably, opinions will be divided, and upon which I will not venture to enter; it is, indeed, a point that requires long experience rather than theory to furnish the answer. The considerations which arise in treating this part of the subject, are similar to those which present themselves when discussing the question as to what expenditure an Insurance Company may make with safety and advantage in founding and extending its business. It is well known what a great difference of opinion exists upon these points.

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#### MATHEMATICAL NOTE.

In the ordinary method of valuing a policy, it is assumed that the premium will be paid each year during the existence of the life assured. But it is quite optional with the assured to keep up the payments, and therefore, in strictness, the annuity consisting of the premiums should be valued by introducing a quantity to denote the probability that the payment of premium will be made in each succeeding year. Thus, let  $q_1$  represent the probability that the premium will be paid at the end of the first year,  $q_2$  the probability that it will also be paid at the end of the second year, and so on; then it will be easily seen that the value of the first year's payment of the annuity of £ will be  $q_1 \frac{D_{m+1}}{D_m}$ , instead of  $\frac{D_{m+1}}{D_m}$ , as in the usual case; the value of the second year's payment will be  $q_1 q_2 \frac{D_{m+2}}{D_m}$ , and so on; whence the value of such an annuity of £1 during the continuance of a life aged  $m$ , which we will denote by  $\bar{a}_m$ , will be

$$\bar{a}_m = \frac{q_1 D_{m+1} + q_1 q_2 D_{m+2} + q_1 q_2 q_3 D_{m+3} + \&c.}{D_m}$$

Again, the value of an assurance of £1 to be paid on the decease of the same life  $m$ , provided the policy is kept in force, is determined in the same way. The value for the first year is  $\frac{l_m - l_{m+1}}{l_m} \cdot r$ , the probability of the Office being exposed to the risk next year is  $q_1$ , and, therefore, the value of £1 to be paid on the decease occurring in the second year, is  $q_1 \frac{l_{m+1} - l_{m+2}}{l_m} \cdot r^2$ , and so on. The total value then is

$$\begin{aligned} \bar{A}_m &= \frac{(l_m - l_{m+1})r + q_1(l_{m+1} - l_{m+2})r^2 + q_1q_2(l_{m+2} - l_{m+3})r^3 + \&c.}{l_m} \\ &= \frac{rD_m - D_{m+1} + q_1(rD_{m+1} - D_{m+2}) + q_1q_2(rD_{m+2} - D_{m+3}) + \&c.}{D_m} \\ &= r \cdot \frac{D_m + q_1D_{m+1} + q_1q_2D_{m+2} + \dots}{D_m} \\ &\quad - \frac{D_{m+1} + q_1D_{m+2} + q_1q_2D_{m+3} + \dots}{D_m} \\ &= r(1 + \bar{a}_m) - \frac{Q_m}{D_m}, \end{aligned}$$

putting  $Q_m$  for the numerator of the second fraction. If now the premium on a policy of £1 be  $P$ , the value of the policy will be  $\bar{A}_m - P\bar{a}_m$ , or

$$r(1 + \bar{a}_m) - \frac{Q_m}{D_m} - P\bar{a}_m.$$

This formula will be inapplicable in practice, because it involves the quantities  $q_1, q_2, \&c.$ , of which the values are not known. An approximation to their values may be obtained in the case of any particular Company, by investigating the number of policies which have been dropped in that Company in the various years of their existence, and assuming that the experience of the future will be the same as that of the past. If we suppose that  $q_1 = q_2 = q_3 = \&c. = 1$ , we get the value of a policy to be

$$r(1 + a_m) - a_m - Pa_m, \text{ or } A_m - Pa_m,$$

(since then  $\frac{Q_m}{D_m}$  becomes  $a_m$ ), which coincides with the usual form.

If the formula above given for the value of a policy were ever to be used in practice for some special object, it would probably be quite sufficient to take  $q_2 = q_3 = \&c. = 1$ , or to assume that if the policy were kept in force for one year it would not afterwards be dropped; in this case the formula becomes very much simplified, and the expression for the value of the policy may be reduced to the form

$$r(1 + q_1a_m) - \frac{D_{m+1}}{D_m} (1 + q_1a_{m+1}) - Pq_1a_m.$$

The above supposes the premium to be just paid: if, however, the premium is just due and unpaid, it is very easily seen that the expression for the value of the policy corresponding to that last given, is simply

$$q_1 \{A_m - P(1 + a_m)\}.$$